ANALYSIS OF THE HEAT TRANSFER DURING FILM EVAPORATION FROM A ROTATING DISK

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On the basis of a semiempirical theory of turbulent transfer, formulas are derived for the mean heat transfer coefficient in the case of film evaporation from a rotating disk.

Film evaporators where a film of evaporating liquid is produced by rotation of a conical or a disk surface have found wide applications in industry [1-4], because of the high rate of heat transfer and the feasibility of evaporating more viscous liquids.

Our earlier studies [5] concerning the heat transfer during evaporation of water and NaCl solutions from a rotating horizontal disk surface have established that $\bar{\alpha}_0$ does not depend on the thermal flux density, within the range of thermal flux densities typical of evaporators, i.e., that heat is dissipated from the hot surface mainly by convection and evaporation from the film surface.

Centrifugal forces contribute to a uniform wetting of a heat transfer surface during rotation when the flow rate of liquid is low, which corresponds to a Reynolds number for the film $Re_f = 10$–50. Considering this, we have assumed in [6] that the film flow is essentially laminar and, using the Kapitsa correction [7] for wave formation, we have obtained a relation for calculating the mean heat transfer coefficient $\bar{\alpha}_1$ in the case of a water film evaporating from a disk:

$$\overline{\mathrm{Nu}}_l = 0{,}42 \frac{\mathrm{Kr}}{1-\left(1-\frac{\mathrm{Kr}}{2}\right)^{\frac{4}{3}}} f\left(\frac{R_w}{R_d}\right) \mathrm{Re}_{\mathrm{film}}^{-\frac{1}{3}}, \tag{1}$$

in dimensionless form with

$$\overline{\mathrm{Nu}}_l = \frac{\bar{\alpha}_l}{\lambda} \cdot \left(\frac{\nu^2}{\omega^2 R_d}\right)^{\frac{1}{3}}; \quad \mathrm{Kr} = \frac{2q\pi\left(R_d^2 - R_w^2\right)}{rG_0}; \quad f\left(\frac{R_w}{R_d}\right) = \frac{1-\left(\frac{R_w}{R_d}\right)^{\frac{8}{3}}}{1-\left(\frac{R_d}{R_d}\right)^2}.$$

Equation (1) is based on the premise that $\bar{\alpha}_0$ decreases with higher wetting rates Γ, namely $\bar{\alpha} \sim \Gamma^{-1/3}$. According to tests in [5], however, $\bar{\alpha}_0$ seems to decrease with increasing Γ_d only at rotational speeds up to $\omega \le 31.4$ sec^{-1}. At $\omega \ge 52$ sec^{-1}, $\bar{\alpha}_0$ becomes independent of Γ_d as the latter varies from $5 \cdot 10^{-3}$ to $3 \cdot 10^{-2}$ kg/m·sec (Fig. 1). This mild effect of the wetting rate on the overall heat transfer coefficient in a centrifugal evaporator has also been noted in [3, 4].

The trend of $\bar{\alpha}_0$ as a function of Γ or Re_f can be explained as follows. The critical value of the Reynolds number Re_f, which characterizes the transition from laminar flow to turbulent flow, varies, according to various authors [8-10], from 60 to 500 and depends on many factors. As is well known [11], external perturbations in a flowing liquid tend to lower the critical value of Re_f. As the disk speed increases, so does the friction at the disk–vapor interface, which in turn amplifies the perturbations in the liquid as the latter flows along the disk. In a study of the flow of two immiscible liquids along a rotating disk, V. A. Yurchenko and A. A. Koptev [12] have established that such liquids begin to mix when $G\omega R/2\pi\nu^2\rho \ge 0.525 \cdot 10^8$ — as is the case in a centrifugal evaporator. This critical value may be said to characterize the beginning of the transition zone between laminar and turbulent film flow.

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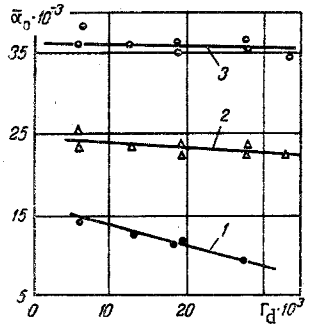

Fig. 1

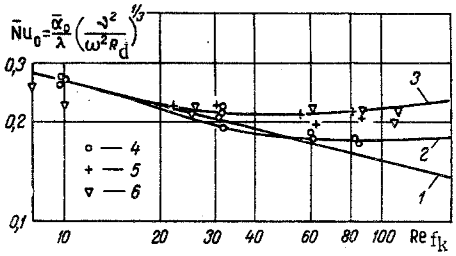

Fig. 2

Fig. 1. Effect of the wetting rate Γ_d (kg/m · sec) and of the rotational speed of a disk on the heat transfer coefficient (W/m^2 · °C) during evaporation of water [5]: 1) $\omega = 20.9$ sec^{-1}; 2) $\omega = 52$ sec^{-1}; 3) $\omega = 104$ sec^{-1}.

Fig. 2. Heat transfer during evaporation of liquid in a film produced on a rotating disk: 1) theoretical curve according to Eq. (1); 2) theoretical curve according to Eqs. (2) and (6); 3) theoretical curve according to Eqs. (2) and (9); test data for water and an NaCl solution: 4) $\omega = 20.9$ sec^{-1}; 5) $\omega = 52$ sec^{-1}; 6) $\omega = 104$ sec^{-1}.

For calculating the heat transfer during evaporation of a film from a rotating disk, we thus require a formula which takes into account the turbulization of the film. It is necessary to consider that, as the disk radius is increased and as Re_f decreases as a result of evaporation, both a turbulent and a laminar zone can, therefore, coexist in a film which flows along the disk. The mean heat transfer coefficient for the entire disk will then be

$$\bar{\alpha}_0 = \bar{\alpha}_l \frac{R_d^2 - R_{cr}^2}{R_d^2} + \bar{\alpha}_t \frac{R_{cr}^2 - R_w^2}{R_d^2}, \tag{2}$$

with $\bar{\alpha}_l$ calculated according to (1) and

$$R_w = R_{cr}, \quad Re_f = Re_{cr} = \frac{G_0 r - q\pi R_{cr}^2}{2\pi R_d \nu \rho r}.$$

A formula for $\bar{\alpha}_t$ can be derived from the semiempirical theory of turbulent transfer in [3]. Since heating at the interface between a vapor and a rotating film constitutes a perturbation which lowers the critical value of Re_f, hence a replacement of g by $\omega^2 R$ in the equations of motion will reflect the difference between film flow along a rotating surface and film flow due to gravity along a stationary surface. The turbulent tangential stress at the wall will then be

$$\tau_s = \omega^2 R \rho \delta. \tag{3}$$

We now introduce the dimensionless thickness

$$\eta_\delta = \frac{v_* \delta}{\nu} = \frac{(\omega^2 R)^{\frac{1}{2}} \delta^{\frac{3}{2}}}{\nu}.$$

According to the two-layer model of turbulent flow proposed by S. S. Kutateladze [14], we have

$$\eta_\delta < 11.6 \text{ (laminar sublayer)},$$

$$\eta_\delta > 11.6 \text{ (turbulent mainstream)}.$$

For $\eta_\delta > 11.6$, the local heat transfer coefficient in the case of film evaporation from a disk is determined from the following equation:

$$\frac{\alpha_t}{\lambda}\left(\frac{\nu^2}{\omega^2 R}\right)^{\frac{1}{3}} = \frac{0.4\mathrm{Pr} \cdot \eta_\delta^{\frac{1}{3}}}{\ln \frac{\sqrt{\eta_\delta} + \sqrt{\eta_\delta - 11.6}}{\sqrt{\eta_\delta} - \sqrt{\eta_\delta - 11.6}} + 4.65\mathrm{Pr}}. \tag{4}$$

With a logarithmic velocity profile, the Re_f number and the dimensionless film thickness during evaporation are related as follows:

$$\mathrm{Re_f} = \frac{G_0 r - q\pi R^2}{2\pi R\nu\rho r} = \eta_\delta(3.0 + 2.5\ln\eta_\delta) - 39. \quad (5)$$

The value of the mean heat transfer coefficient in the turbulent zone $\bar{\alpha}_t$ is found by integrating $\bar{\alpha}_t$ over the surface, with $(F/\pi)^{1/2}$ replacing R in Eqs. (4) and (5):

$$\bar{\alpha}_t = \frac{1}{F_{cr} - F_w}\int_{F_w}^{F_{cr}} \alpha_t \, dF. \quad (6)$$

Both $\mathrm{Re_{cr}}$ and thus also F_{cr} can be evaluated from the expression

$$\frac{G_0 r - q\pi R_{cr}^2}{2\pi R_{cr}\nu\rho r} = 11.6(3.0 + 2.5\ln 11.6) - 39. \quad (7)$$

According to the three-layer model of turbulent flow, we have

$\eta_\delta < 5$ (laminar sublayer),

$5 < \eta_\delta < 30$ (transition zone),

$\eta_\delta > 30$ (turbulent mainstream).

The relation between $\mathrm{Re_f}$ and η_δ within the range $5 < \eta_\delta < 30$ is

$$\mathrm{Re_f} = 5\eta_\delta \ln\eta_\delta - 8.05\eta_\delta + 12.5, \quad (8)$$

according to Protalski [15]. Here $\eta_\delta = 30$ corresponds to $\mathrm{Re_f} = 281$ and, therefore, centrifugal evaporators fall well within this range $5 < \eta_\delta < 30$. In this case, with the aid of I. V. Lomanskii's solution for the transition zone [16] and with $(F/\pi)^{1/2}$ replacing R, we have

$$\bar{\alpha}_t = \frac{1}{F_{cr} - F_w}\int_{F_w}^{F_{cr}} \lambda \left[\frac{\nu^2}{\omega^2\left(\frac{F}{\pi}\right)^{\frac{1}{2}}}\right]^{-\frac{1}{3}} \frac{\mathrm{Pr}\cdot\mathrm{Re_f^{0.2}}}{5\mathrm{Pr} + 2.9\mathrm{Pr}^{0.33}\mathrm{Re_f^{0.2}}} \, dF. \quad (9)$$

In our case R_{cr} and F_{cr} will be determined from (8) with $\eta_\delta = 5$.

Test data according to [5] and values calculated according to Eqs. (2), (6), and (9) are shown in Fig. 2. The test values for the mean heat transfer coefficient in the case of film evaporation from a rotating disk surface are based on water and up to 8% NaCl solutions under atmospheric pressure. Under these conditions and with $\omega = 20.9\ \mathrm{sec}^{-1}$,

$$\frac{G_0}{2\pi R_d \nu\rho} \cdot \frac{\omega R_d^2}{\nu} < 0.8 \cdot 10^8,$$

and, according to Fig. 2, Eq. (1) yields a satisfactory agreement with test values, the latter rising higher above the theoretical curve with increasing values of $\mathrm{Re_f}$. The relations derived with film turbulization taken into account describe the process of heat transfer during film evaporation from a rotating disk.

NOTATION

ω	is the angular velocity of a disk, sec^{-1};
q	is the thermal flux density, $\mathrm{W/m^2}$;
ν	is the kinematic viscosity, $\mathrm{m^2/sec}$;
ρ	is the density, $\mathrm{kg/m^3}$;
λ	is the thermal conductivity, W/m · °C;
r	is the heat of evaporation, kJ/kg;
G_0	is the initial flow rate of the liquid, kg/sec;
R	is the radial distance from the disk center, m;
R_w	is the radius to which the disk has been wetted, m;
R_d	is a finite disk radius;
R_{cr}	is the critical radius;
$\Gamma = G/2\pi R\nu\rho$	is the wetting rate, kg/m · sec;
Γ_d	is the wetting rate referred to a finite disk radius, kg/m · sec;
$F = \pi R^2$	is the disk surface, $\mathrm{m^2}$;
$\mathrm{Re_f} = \Gamma/\nu\rho$;	

$Re_{f,d} = (G_0 r - q\pi R_d^2)/2\pi R_d \nu\rho r$;

v_*	is the average turbulent velocity, m/sec;
δ	is the film thickness, m;
$\bar{\alpha}_0$	is the mean heat transfer coefficient for the entire disk, $W/m^2 \cdot °C$;
$\bar{\alpha}_l$	is the mean heat transfer coefficient for the laminar zone of film flow;
$\bar{\alpha}_t$	is the mean heat transfer coefficient for the turbulent zone of film flow.

LITERATURE CITED

1. A. V. Viter, A. N. Zaichenko, and V. P. Plavnik, Khim.-Farmatsevt. Zh., No. 12 (1968).
2. K. C. Hickman, V. D. Hogan, D. U. Eibling, and V. L. Buckel, in: Distillation of Saline Water [Russian translation], IL, Moscow (1963).
3. K. C. D. Hickman, Industr. Engrg. Chem., 49, 786 (1957).
4. L. A. Bromley, R. F. Humphreys, and W. J. Murray, Trans. ASME, Heat Transfer, 88C, 87 (1966).
5. A. I. Butuzov and V. G. Rifert, Izv. VUZov SSSR, Mashinostroenie, No. 2 (1972).
6. A. I. Butuzov and V. G. Rifert, in: Thermophysics and Heat Engineering, Vol. 19 [in Russian], Izd. Naukova Dumka, Kiev (1971).
7. P. L. Kapitsa, Zh. Éksper. i Teor. Fiz., 18, No. 1 (1948).
8. W. Brotz, Chemie-Ingenieur Technik, Nos. 8-9, 470 (1954).
9. L. Ya. Zhivaikin, Khim. Mashinostroenie, No. 6 (1961).
10. S. I. Friedman and Miller, Industr. Engrg. Chem., 33, 885 (1941).
11. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Izd. Nauka, Moscow (1970).
12. V. A. Yurchenko, A. A. Koptev, A. I. Zaitsev, A. K. Zhobrovskii, and Ya. S. Yanov, Teoret. Osnovy Khim. Tekhnolog., 3 (1969).
13. T. V. Karman, in: Problems in Turbulence [Russian translation], Izd. ONTI (1936).
14. S. S. Kutateladze, Fundamentals of Heat Transfer Theory [in Russian], Izd. Nauka, Novosibirsk (1970).
15. S. Portalski, Chem. Engrg. Sci., 18, 787 (1963).
16. I. V. Domanskii and V. N. Sokolov, Zh. Prikl. Khim., 11, No. 1 (1967).